London Equation

Dr. K.U. Madhu,

Asst. Prof., Dept. of Physics,

S.T. Hindu College, Nagercoil

London Equations

London Brothers (F.London and H.Londan) proposed addition of two more equations to Maxwell's equation to explain Meissner effect.

Super conductor is composed of two distinct types of electrons, normal electrons and super electrons. Normal electrons behave in usual manner, Super electrons behave in different way, they experience, No Scattering

Above T_C, all the electrons are normal.

Below T_C, with decrease of temperature increasing electrons become super electrons.

At T = 0K, all are super electrons.

∴ The conduction electron density n,

$$n = n_n + n_s$$

n_n → density of normal electrons.

n_s → density of super electrons.

The Normal current and the super current are assumed to flow parallel.

As the super current flows with no resistance, it carries the entire current induced by any small transitory electric field E. While current due to normal electrons is negligible.

If V_s is average velocity of super electrons, m is its mass, and e is its charge, then the equation of motion.

$$m\frac{dv_s}{dt}=-eE \quad \text{ie.,} \quad \frac{dv_s}{dt}=-\frac{eE}{m}$$

Current density of super electrons in

$$J_s=-en_s v_s$$

$$\therefore \frac{dJ_s}{dt}=-en_s \frac{dv_s}{dt}=-en_s.\left(\frac{-eE}{m}\right)$$

$$\frac{dJ_s}{dt}=\frac{n_s e^2}{m}E. \qquad ..(5.1)$$

This is called First London equation,

For $E=o$, $\frac{dJ_s}{dt}=o$ ie., J_s is constant or vice – versa.

The corresponding normal current density is

$J_n=\sigma E \quad E=0$, leads to another important result when combined with the maxwell's equation,

$$\Delta \times E = -\left(\frac{dB}{dt}\right) \qquad ..(2)$$

$$\text{ie} \frac{dB}{dt} = o \;\; or \;\; B = const$$

Since this contradicts with the Meissner effect, London proposed some modification to remove this discrepancy

Taking curl on equation (1) we get

$$\Delta \times \frac{dJ_s}{dt} = \frac{n_s e^2}{m} (\Delta \times E)$$

Substituting equation (2) we get

$$\Delta \times \frac{dJ_s}{dt} = -\frac{n_s e^2}{m} . \frac{dB}{dt}$$

Integrating w. r. to time and put zero for constant of integration, we get

$$\Delta \times J_s = -\frac{n_s e^2}{m} B. \qquad ....(3)$$

This is called Second London equation. This is in agreement with the experiment.

Penetration Depth

According to London equation the magnetic flux decreases exponentially (ie., the flux does not drops to zero at the surface of Type I superconductors).

To calculate to what depth the flux penetrates.

Mexwell's equation, $$\Delta \times B = \mu_0 J_s$$

Taking curl on both sides, $$\Delta \times \Delta \times B = \mu_0 (\Delta \times J_s)$$

$$\Delta (\Delta \cdot B) - \Delta^2 B = \mu_0 (\Delta \times J_s)$$

$$\Delta \cdot B = 0, \quad \therefore -\Delta^2 B = \mu_0 (\Delta \times J_s)$$

We have $\Delta \times J_s = -\frac{n_s e^2}{m} . B$

$$\therefore \Delta^2 B = \mu_0 \frac{n_s e^2}{m} . B$$

$$\Delta^2 B = \frac{B}{\lambda^2} \qquad ..(4)$$

Where $\lambda = \left(\frac{m}{\mu_0 n_s e^2} \right)^{1/2}$ is called London Penetration depth.

The solution of the equation (4) is

$$H = H(0)\exp\left(\frac{-x}{\lambda}\right) \qquad ...(5)$$

Graphical representation of equation (5) is shown in Figure

Ie., The flux density decreases exponentially inside the Superconductor.

To define penetration depth λ,

Let $x = \lambda$, $\therefore \frac{H}{H(0)} = e^{-1}$

$$H = \frac{H(0)}{e^{-1}} \qquad ...(6)$$

$\therefore$ The penetration depth λ can be defined as the depth from the surface at which the magnetic flux density falls to $\frac{1}{e}$ of its initial value at the surface.

The penetration depth is about 500 Å

With respect to temperature

$$\lambda(T) = \lambda(0)\left[1 - \frac{T^4}{T_c^4}\right]^{-1/2}$$

$\lambda(0)$ is the penetration depth at T = 0K.

λ increases with temperature T and at $T = T_c$, $\lambda = \infty$