BCS THEORY

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BCS Theory

BCS theory of Superconductivity was introduced by Bardeen, cooper and Schrieffer in 1957. This theory helps to explain zero resistivity, Meissner effect, isotope effect etc.,

(i) Electron – Electron interaction via lattice Deformation:

When an electron passing through the packing of positive ions, the electron is attracted by the neighbouring positive ions, form a positive ion core as shown in Figure 5.10 and get screened by them. This screening reduces the effective charge of this electron and the ion core may produce a net positive charge on this assembly. Due to the attraction between the electron and the ion core the lattice gets deformed. This deformation is greater for smaller mass of positive ion core.

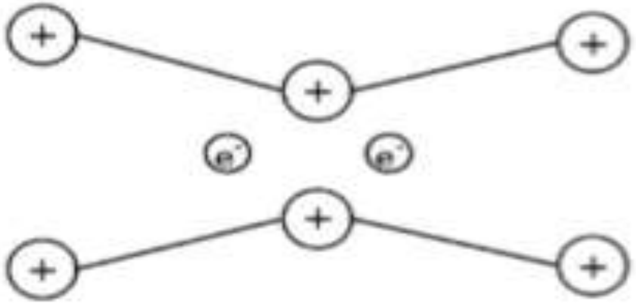
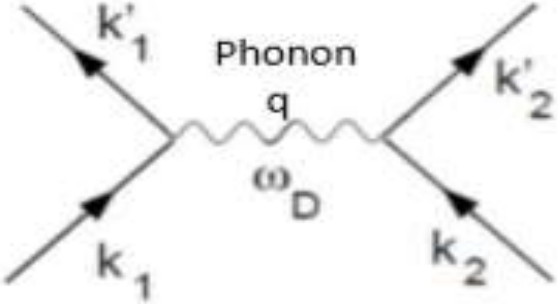


Fig. 5.10 *Cooper pair of electrons.*

If another electron passes by the side of the assembly of the electron and the ion core, it gets attracted towards the assembly (Fig. 5.10). By this way the second electron interacts with the first electron via lattice deformation. This interaction is due to the exchange of a virtual phonon q, between the two electrons. This interaction process can be written in terms of wave vector K,

$K_1 - q = K_1'$ *and* $K_2 + q = K_2'$ ie., $K_1 + K_2 = K_1' + K_2'$



∴ The net wave vector of the pair is conserved. The momentum is transferred between the electrons. These two electrons together form a cooper pair and is known as cooper electron. This process leaves the lattice invariant.

Cooper pair:

To study the mechanism of cooper pair formation, consider the Fermi – Dirac distribution function of electrons in metals at absolute zero

$$F(E) = \frac{1}{\exp\left(\frac{E - E_F}{kT}\right) + 1}$$

At T = 0k all the quantum states below the Fermi level E_F are completely filled and all the quantum states above E_F are completely empty (fig.)

Since all the quantum states with energies $E \leq E_F$ are filled, by Pauli's exclusion principle they are forced to occupy states having energies $E > E_F$. Cooper showed that if there is an attraction between the two electrons, they are able to form a bound state so that their total energy is less than $2E_F$.

These electrons are paired to form a single system and their motions are correlated. These two electrons together form a cooper pair and is known as cooper electron.

The binding is strongest when the electrons forming the pair have opposite momenta and opposite spins, $K\uparrow, -K\downarrow$.

If there is an attraction between any two electrons lying in the neighbourhood of the Fermi surface then all other electrons lying in that region will form cooper pairs. These pairs of electrons are superelectrons which are responsible for the superconductivity.

BCS Ground State:

In normal metals, the excited states lie just, above the Fermi surface. Small excitation energy is sufficient to excite an electron from Fermi surface to excited state.

In super conducting materials, when a pair of electron lying just below the Fermi surface is taken just above it, they form a cooper pair. This continues until the system can gain no additional energy by pair formation and hence the total energy is reduced.

Important features of BCS ground state:

1. Even At absolute zero, the energy distribution of electrons does not show any abrupt discontinuity as in normal metals

2. The states are occupied in pairs, ie., A cooper pair is imagined to be an electron pair in which the two electrons always occupy states and so on with opposite k- vectors and spins.

Flux Quantization :

Ginburg and Landua developed a macroscopic theory for superconducting phase transition based on thermodynamics in 1950.

To describe superconducting state, Ginzburg and Landau introduced a complex wave function ψ. It is an order parameter, and is a function of position in the material, ie., it is not constant and vanishes above T_c

The density of the superconducting electrons $n_s \alpha |\psi|^2$

ψ can be written in terms of magnitude and phase as $\psi = |\psi| \exp(i\varphi)$.

Then the current density $J = -\left[\frac{2e^2}{mc} A + \frac{e\hbar}{m} \nabla\varphi\right] |\psi|^2$

$A \rightarrow$ is vector potential.

Consider a ring shaped super conducting material and take a closed path (Fig. 5.12)

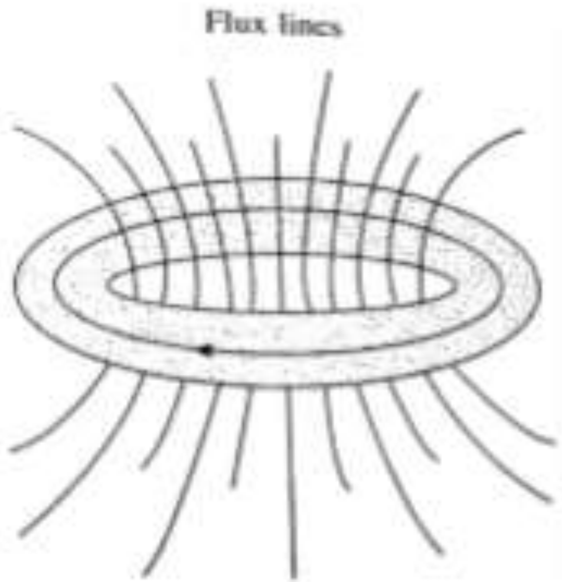


Fig. 5.12 A ring shaped superconducting material showing the path of integration C through its interior

$$\oint J.dl = |\psi|^2 \oint \left(\frac{2e^2}{mc} A + \frac{e\hbar}{m} \nabla \varphi \right).dl = 0 \qquad ...(1)$$

According to stokes theorem

$$\int J.dl = \int \nabla \times A.ds = \int_s B.ds = \phi \qquad ..(2)$$

ϕ -is the flux enclosed by the ring.

Since the order parameter is single valued, the phase change around the closed path must be zero or integral multiple of 2π .

$$\text{ie} \quad \oint \nabla \phi .dl = 2\pi n, \qquad n = 0,1,2,.... \qquad (3)$$

Substituting (2) & (3) in (1) and solving for ϕ

We get

$$\phi = \frac{nhc}{2e} = n\,\phi_0$$

Where $\phi_0 = \frac{hc}{2e} = 2.07 \times 10^{-5}$ weber is known as fluxoid or flux quantum. Thus the magnetic flux enclosed by the ring is quantized.

The flux through the ring is the sum of the flux due to the external source and the flux due to the super current flowing through the ring.