Antiferromagnetic order

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In an antiferromagnet the spins are ordered in an anti parallel arrangement with zero net moment at temperatures below the Neel temperature. The susceptibility of antiferromagnet is not infinite at $T = T_N$.

Antiferromagnet is a special case of a ferromagnet for which both sub lattices A and B have equal saturation magnetizations. The Neel temperature in the mean field is given by $T_N=\mu c$ where c refers to single lattice. The susceptibility in the paramagnetic region $T> T_N$ is obtained as

$$\frac{2c}{T+\mu c} = \frac{2c}{T+TN}$$

The value of $\frac{\theta}{T_N}$ of the observed magnitude may be obtained when next – nearest – neighbor interactions are provided for, and when possible sublattice arrangements are considered. If a mean field constant $-\varepsilon$ is introduced to describe interactions within a sub lattice, then $\frac{\theta}{T_N} = \frac{\mu+\varepsilon}{(\mu-\varepsilon)}$.

Susceptibility below the Neel temperature

There are two situations; with the applied magnetic field perpendicular to the axis of the spins; and with the field parallel to the axis of the spins. At and above the Neel temperature the susceptibility is nearly independent of the direction of the field relative to the spin axis.

For B_a perpendicular to the axis of the spins, the susceptibility is $\chi_{perpendicular} = \frac{2M\varphi}{B_a}$

In the parallel orientation, the magnetic energy is not changed, thus the susceptibility at T= oK is zero $\chi_{parallel} = 0$.

The parallel susceptibility increases with temperature up to T_N .in very strong fields the spin system will turn discontinuously from the parallel orientation to the perpendicular orientation where the energy is lower.

Magnons in antiferromagnets

Consider a one dimensional antiferromagnetic crystal. The spins are aligned alternatively up an down. Let,

2P even numbered spins have $S^z = S$.

(2p+1) odd numbered spins have $S^z = -S$

The spins interact with the nearest neighbors only,ie., p^{th} spin interacts with $(P+1)^{th}$ spin and $(P-1)^{th}$ spin. In the anti ferromagnetic crystal, Heisenberg exchange interaction is $-2J_{12}\sum s_1 s_2$, where J is negative.

The Cartesian form of the relation between the rate of change of angular momentum and the torque is

$$\frac{ds_p{}^x}{dt} = \frac{2J}{\hbar}\left[s_p{}^y\left(S^z_{p-1} + S^z_{p+1}\right) - s_p{}^z\left(S^y_{p-1} + S^y_{p+1}\right)\right] \ldots\ldots\ldots\ldots(1)$$

$$\frac{ds_p{}^y}{dt} = \frac{2J}{\hbar}\left[s_p{}^z\left(S^x_{p-1} + S^x_{p+1}\right) - s_p{}^x\left(S^z_{p-1} + S^z_{p+1}\right)\right] \ldots\ldots\ldots\ldots(2)$$

$$\frac{ds_p{}^z}{dt} = 0 \ldots\ldots\ldots\ldots(3)$$

Substituting the special conditions in these eqns;

For even numbered A spins $S^z = S$

For odd numbered B spins $S^z = -S$

We wil get two sets of equations, one set for A spins 2p atoms and other set for B spins (2p+1) atoms . For A spins, in eqns (1) and (2)

Put, $S^z_{2p} = S$; S^z_{2p+1} and $S^z_{2p-1} = -S$; P = 2p, P+1 = 2p+1, P-1= 2p-1.

$$\frac{ds_{2p}{}^{x}}{dt} = \frac{2J}{\hbar}\left[S^{y}_{2p}\left(S^{z}_{2p-1} + S^{z}_{2p-1}\right) - S^{z}_{2p}\left(S^{y}_{2p-1} + S^{y}_{2p-1}\right)\right]$$

$$\frac{dS^{x}_{2p}}{dt} = \frac{2JS}{\hbar}\left[-2s_{2p}{}^{y} - S^{y}_{2p-1} - S^{y}_{2p+1}\right] \ldots\ldots\ldots\ldots\ldots(4a)$$

Eqn (2) becomes,

$$\frac{dS^{y}_{2p}}{dt} = \frac{-2JS}{\hbar}\left[-2s_{2p}{}^{x} - S^{x}_{2p-1} - S^{x}_{2p+1}\right] \ldots\ldots\ldots\ldots\ldots(4b)$$

For B atoms, in eqns (1) (2) substitute $S^{z}_{2p} = S^{z}_{2p+2} = S; S^{z}_{2p+1} = S^{z}_{2p-1} = -S, P = 2P + 1$

Eqn (1) becomes,

$$\frac{dS^{x}_{2p+1}}{dt} = \frac{2JS}{\hbar}\left[2S^{y}_{2p+1} + S^{y}_{2p} + S^{y}_{2p+2}\right] \ldots\ldots\ldots\ldots\ldots(5a)$$

Eqn (2) becomes,

$$\frac{dS^{y}_{2p+1}}{dt} = \frac{-2JS}{\hbar}\left[2S^{x}_{2p+1} + S^{x}_{2p} + S^{x}_{2p+2}\right]\ldots\ldots\ldots\ldots\ldots(5b)$$

If $S^{x} + i\,S^{y} = s^{+}$, $\frac{ds^{+}}{dt} = \frac{ds^{x}}{dt} + i\frac{ds^{y}}{dt}$

$$\frac{ds^{+}_{2p}}{dt} = \frac{ds^{x}_{2p}}{dt} + i\frac{ds^{y}_{2p}}{dt}\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

Substitute eqn (4a) and (4b) and simplify the eqn (6)

$$\frac{dS^{+}_{2p}}{dt} = \frac{2iJS}{\hbar}\left[2S^{+}_{2p} + S^{+}_{2p-1} + S^{+}_{2p+1}\right] \quad \text{.................(7a)}$$

$$\frac{dS^{+}_{2p+1}}{dt} = -\frac{2iJS}{\hbar}\left[2S^{+}_{2p+1} + S^{+}_{2p} + S^{+}_{2p+2}\right] \quad \text{.................(7b)}$$

The solutions of eqn (7a) and (7b) are

$$S^{+}_{2p} = U\exp[i2pka - i\omega t] \quad \text{..................................(8a),}$$

$$S^{+}_{2p+1} = V\exp[i(2p+1)ka - i\omega t] \quad \text{...... ....(8b)}$$

On substituting eqn(8a) and (8b) in eqn(7a) and (7b), we get,

$$\omega U = \frac{1}{2}\omega_{ex}[2U + V\exp(-ika) + V\exp(ika)] \quad \text{..........(9a)}$$

$$-\omega V = \frac{1}{2}\omega_{ex}[2V + U\exp(-ika) + U\exp(ika)] \ldots\ldots\ldots\ldots (9b)$$

Where $\omega_{ex} = \frac{-4JS}{\hbar}$

Eqns (9a), (9b) have a solution when $\begin{vmatrix} \omega_{ex} - \omega & \omega_{ex}(\cos ka) \\ \omega_{ex}(\cos ka) & \omega_{ex} + \omega \end{vmatrix} = 0$

$$\omega_{ex}{}^2 - \omega^2 = \omega_{ex}{}^2\cos^2 ka$$

$$-\omega^2 = \omega_{ex}{}^2\cos^2 ka - \omega_{ex}{}^2$$

$$\omega^2 = \omega_{ex}{}^2(1 - \cos^2 ka)$$

$\omega = \omega_{ex}|\sin ka| \ldots\ldots\ldots\ldots (10)$. This is the dispersion relation for anti ferromagnetic material.

If $ka \ll 1$, then $\sin ka = ka$.

The dispersion relation becomes $\omega = |ka|$.